Domain walls, wakes, and structure formation

Alessandro Massarotti and Jean M. Quashnock
Enrico Fermi Institute, The University of Chicago, Chicago, Illinois 60637
and NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, Illinois 60510-0500
(Received 22 June 1992)

We consider a cosmological domain wall model where the dark matter interacts nongravitationally with the walls. We find that the dark matter is quickly (as early as $z \simeq 100$) swept up in wakes and voids, with the wakes growing with time in a self-similar manner. The dark matter wakes, of order $5h^{-1}$ Mpc thick today, tug on the baryons, which also trail behind in wakes of similar thickness. We find that the peculiar motions induced are of the order of a few hundred km/s, coherent on a scale of order $20h^{-1}$ Mpc. The baryon overdensities generated by the domain walls are distributed in a geometry similar to that of galaxy superclusters found in recent surveys.

PACS number(s): 98.80.Cq, 95.35.+d, 98.80.Hw

I. INTRODUCTION

One of the most interesting unsolved problems of modern cosmology is the formation of structure and the growth of clustering, especially on very large scales. Recent surveys have found statistical correlations between galaxies, even up to scales as large as $40h^{-1}$ Mpc [1]. The "pencil beam" deep surveys appear to find numerous walls of galaxies, at typical intervals of $50-100h^{-1}$ Mpc [2]. Structures on scales bigger than $10h^{-1}$ Mpc are very difficult to incorporate in the simplest version of the cold dark matter (CDM) model [3], which involves the growth, via gravitational instability, of primordial adiabatic perturbations with a scale-invariant power-law spectrum.

An independent problem facing any perturbation model is the very low experimental upper limit to the cosmic microwave background radiation (CMBR) temperature fluctuations at the 1° angular scale, found by the MAX group [4] to be $\Delta T/T < 3\times 10^{-5}$. This angular scale corresponds roughly to a $100h^{-1}$ Mpc comoving scale, at recombination. In the framework of perturbation models, it is difficult to obtain the observed galaxy correlations at large scales, given the small value of density fluctuations inferred from the CMBR distortion limits. Furthermore, the peculiar velocity field, again on scales of about $50-100h^{-1}$ Mpc, seems to be inconsistent with the standard CDM model [5].

In recent years, an alternative class of models for galaxy and structure formation has been developed, based on topological defects, which arise from a cosmological phase transition. One of these is the cosmic strings model. Strings, arising from a phase transition at the grand unified theory (GUT) energy scale, would indeed have the right range of energy density, although not necessarily the right dynamics, to act gravitationally on the surrounding matter, generating matter inhomogeneities and collapse [6]. Recently, textures have also been studied extensively [7].

Phase transitions can also produce domain walls. However, if the transition takes place at high-energy scales, such as the GUT, the electroweak, or the quark-hadron phase transition scales, domain walls are a disaster from a cosmological point of view. Their energy density is far too big and dominates the present energy density of the Universe. There are many reasons why that cannot be true [8].

Domain walls can work, however, in the context of late-time phase transitions. In the schizon model [9], for example, domain walls can arise from a phase transition taking place well after primordial nucleosynthesis. Their energy density could be small enough to be consistent with the CMBR observations. However, the topic of a natural particle physics model generating a late-time phase transition is far from settled. Our view is that, before comparing detailed Lagrangian models, one should ensure that domain walls can be of any interest from a dynamical point of view. In fact, the simplest domain wall models fail to generate large-scale structure, since the typical interwall distance, or network scale, grows too rapidly with the horizon [10,11].

Domain walls do become cosmologically interesting when some *nongravitational* interaction between these defects and the dark matter is introduced in the study of wall dynamics. The general issue of wall-particle interaction was discussed, for example, by Everett and by Kibble [12], but not in connection with late-time phase transitions. This interaction can occur because of a Lagrangian coupling term between the field associated with the domain walls and other fields. This sort of coupling is commonly present both in axion models, e.g., Huang and Sikivie [13], and the schizon models mentioned above.

Reference [14] gives a more specific discussion of the dynamical implications of wall-particle interaction for walls arising during late-time phase transitions. There, a wall-matter coupling of Yukawa type is assumed, in analogy with axion and schizon models. Reference [14] assumes that the scalar field of the walls is coupled with a dark matter component since, in general, late-time phase transitions are possible only for weakly interacting fields. If the wall-particle coupling is strong enough, the resulting effects can be quite interesting, since particles passing

through the region where the classical value of the scalar field changes (this is the region that defines the domain wall) will be reflected. Wall-particle scattering leads to momentum exchange and therefore friction when the wall moves through the dark matter with a given relative velocity. In conditions of high friction, the domain walls are slowed down to very nonrelativistic speeds, and move in an overdamped regime similar to that studied in condensed-matter physics [15].

The dark matter is pushed by the walls in the regions through which they move, giving rise both to a strong underdensity behind the walls and to a strong overdensity in front of them. Hence, one is left with sheetlike structures, such as wakes and associated voids, which follow the pattern of the domain wall network. The sheets of overdensity in the dark matter gravitationally drag the baryonic component of the matter. Thus, even though they do not interact directly with the domain walls, baryons are affected indirectly, via gravitation, by the wakes induced in the dark matter component. Along these lines, one of us discussed analytically the range of parameters that are interesting if one wants the domain wall network to be associated with the observed "great walls" [16]. The CMBR distortions produced by the gravitational effects of walls and wakes are small, consistent with the present limits and observations [17].

The issue of the overdamped wall mode which is most difficult to approach analytically is the role of gravity in the evolution of the wakes. This is especially true when dealing with the gravitational effects of the wakes on the baryonic component. For this reason, in this paper we study the model numerically, and we look at the process of wake formation in detail. The very important issue treated here is if, when, and exactly where the baryonic gravitational collapse occurs. We find that our domain wall model is complementary to the standard CDM model by providing a new mechanism for the formation of structure on a very large scale. The paper is structured as follows: in Sec. II, we summarize the domain wall scenario introduced in our previous work; in Sec. III, we describe the fluid-dynamical equations we have used; in Sec. IV, we discuss the numerical code; in Sec. V, we give our results; in Sec. VI we conclude our discussion by relating the results to observations.

II. THE DOMAIN WALL SCENARIO

Domain walls arise when a scalar field has a potential with two or more disconnected minima. As an example, such is the case for the quartic potential of a real scalar field $V(\Phi)=\lambda^2(\Phi^2-\Phi_0^2)^2$. In all of what follows we limit ourselves to models in which the potential minima are degenerate. In such cases, the domain wall network stretches solely under the driving force of its surface tension [10,11]. In general, wall networks are very complicated curved surfaces, whose topology is determined by the number and relative depth of the potential minima.

Independently of the details of the Lagrangian, we can identify some general features. Since the driving force of the walls is surface tension, these can be locally described by the following equation of motion:

$$\gamma^2\dot{v}+3\left[\frac{\dot{a}}{a}\right]v+\frac{P_f}{\sigma\gamma}=-\frac{1}{\tilde{R}}\ . \tag{1}$$

In this expression v is the physical speed of the infinitesimal wall element considered (in units of c), the velocity being normal to surface itself. Also, $\gamma=(1-v^2)^{-1/2}$ and a is the universal scale factor. In what follows, we will take $a=(t/t_0)^{2/3}$, implying that all our calculations are performed in an expanding universe with negligible curvature ($\Omega=1$ within the range of redshifts examined). The surface energy density of the walls, σ, depends solely on the field Lagrangian, while $1/\tilde{R}$ is the local value of the scalar curvature of the wall. Finally, P_f is the friction pressure exerted by the dark matter on the wall.

Let us now examine Eq. (1) in more detail. The first term is the relativistic generalization of Newton's second law, modified to account for the universal expansion by introducing the damping term $3(\dot{a}/a)v$. The right-hand side (RHS) is the surface tension, which is proportional to the curvature. P_f accounts for the friction that arises when the wall moves relative to the surrounding dark matter, and it has units of pressure. When the P_f term dominates over the other noninertial terms of Eq. (1), the motion of the wall is friction dominated, and its speed is $v \ll 1$. In the frictionless regime, however, the motion is relativistic. P_f is proportional to the particle-wall collisions rate and to the average momentum exchange per collision. These quantities, in turn, depend on the temperature T of the dark matter, on the mass m of the dark matter particles, and on the reflection coefficient of the domain wall [14]. For example, if the reflection coefficient is unity, one can identify two friction regimes.

(1) *"Hot" gas regime.* In this case, the wall moves through the dark matter with a speed v much smaller than the average thermal speed of the particles, $v \ll \bar{v}_{\rm th}$. The average momentum exchange is $\Delta p \sim T$. The wall collides with the matter at almost equal rates from both sides, and the differential collision rate between the back and the front of the wall is $\sim T^3 v$. Therefore, $P_f \simeq vT^4$. The motion of the wall causes an overdensity in front of it, but the differential of particle density between the back and the front of the wall is small, namely, $\delta\rho/\rho \sim v/\bar{v}_{\rm th}$.

(2) *"Cold" gas regimes.* It takes place when $v \gg \bar{v}_{\rm th}$. The momentum exchange is $\Delta p = mv$ and the collision rate is $v\rho_D/m$, where m is the mass of the dark matter particles and ρ_D is the dark matter energy density. Hence, we get $P_f \sim \rho_D v^2$. Walls create voids behind and wakes ahead. It is important to note that walls produce large inhomogeneities in the dark matter only when they are in this regime.

In a previous study [18], we considered the other cases in which the wall-particle reflection coefficient is less than unity and energy dependent. There are thus a variety of wall-matter interaction scenarios that can be considered. In what follows, we will concentrate on the case in which this coefficient is unity.

In this case, the friction is most effective at early times, in both "hot" and "cold" regimes. In our work, we start with the assumption that the domain walls are indeed

able to generate big inhomogeneities in the dark matter on scales of $50-100h^{-1}$ Mpc. We consider this as the range for the scale of our wall network. These assumptions mean that the walls today are moving through a "cold" gas and in conditions of very effective friction. By simple inspection of the redshift dependence of P_f and by using Eq. (1), it can be seen that if the friction is dominant today then it always was; hence, the network started and maintained its present configuration and the comoving scale of the network is roughly constant. By causality considerations [17] one argues that the phase transition must have taken place at or after recombination, at redshifts less than a few thousand, since the comoving scale of the network has to be smaller than the horizon scale at the time of the phase transition. At the transition, the physical scale of the network is of the same order as the wall thickness Δ and the correlation length of the field. Thus, we obtain a lower limit on Δ of a few kpc. Such a range of parameters describes walls originating in typical late-time phase transitions.

In all scenarios, galaxy formation will take place only when the walls move in the "cold gas regime," since only then are big inhomogeneities (voids and wakes) in the dark matter produced. It is therefore of key importance to know the details of the wake formation and evolution in this regime. In particular, we would like to determine the resulting density profile for both dark matter and the baryons. We are especially interested in extracting the value of redshift z when the baryons collapse. If domain walls are responsible for the formation of sheets and voids, the scale of the network should be $50-100h^{-1}$ Mpc and the thickness of the wakes should be of the order of a few Mpc. These are the only relevant constraints on our numerical study of wake formation.

In our study, we restrict ourselves to the examination of the overdamped cold gas regime, which is the most important, from the point of view of baryon collapse. We begin our simulation at the time $t_{\rm in}$, with corresponding scale factor $a_{\rm in} \gtrsim 10^{-3}$ (corresponding to redshifts $z_{\rm in} \lesssim 1000$), when the cold gas regime starts. Equation (1) allows us to calculate the motion of a wall section of given curvature. In the friction-dominated (overdamped) case the first two terms are small, and one finds

$$\frac{K\tilde{\rho}_D v^2}{\sigma} = -\frac{1}{\tilde{r}} , \tag{2}$$

where the relevant quantities are expressed in comoving coordinates, namely, $v = v/a$, $\tilde{r} = \tilde{R}/a$, and $\tilde{\rho}_D = a^3 \rho_D$. Now, $K=2$, if the walls were only scattering the dark matter particles once. This would mean that the wall comoving speed is constant in time. In fact, our numerical simulation shows that the domain wall scatters the dark matter several times, and while the speed of the particle decays due to the universal expansion, the wall comoving speed v remains about constant. Such an effect has been examined already in our previous work. K has to be modified to take into account multiple collisions, namely $K \simeq 2.7$, as we will discuss in Sec. IV. We find $v = 5\times 10^{-3} \Omega_D^{-1/2} (\sigma/10^{-1}\ \ \mathrm{MeV}^3)^{1/2} (\tilde{r}/100h^{-1}\ \mathrm{Mpc})^{-1/2}$, Ω_D is the ratio of the dark matter to the critical density.

Note that, for a given infinitesimal wall section under consideration, the radius of curvature $\tilde{r}$ remains virtually constant in time, since the whole configuration is basically frozen in. The network itself is composed of segments whose curvature is distributed with a fairly broad dispersion around a mean value [10]. In the following, we will study the average characteristics of the network, by considering a wall section with typical comoving curvature. Since the average radius of curvature is very large (of order $50h^{-1}$ Mpc), we will treat the wall as essentially flat, moving with a constant comoving speed which is governed by the local curvature. In the final section, we will consider variations in the local curvature. Such variations have important ramifications for structure formation.

III. HYDRODYNAMICAL EQUATIONS

We performed a one-dimensional computer simulation of a moving straight wall, studying the details of wake formation in the case where the wall moves through matter in the cold gas regime. As we have previously discussed, this is the only regime where analytical estimates are not detailed enough to fully understand the evolution of the system. We limited the simulation to one spatial dimension and one wall, since the interwall distance is much larger than the thickness of the inhomogeneities created by the motion of the wall.

The equations describing the dark matter and the baryons are the usual fluid equations, applied to an expanding universe [19]:

$$\frac{\partial \mathbf{v}_D}{\partial t} + \frac{1}{a}(\mathbf{v}_D \cdot \nabla)\mathbf{v}_D + \frac{\dot{a}}{a}\mathbf{v}_D = -\frac{1}{\rho_D a}\nabla p_D - \frac{1}{a}\nabla\Phi_G - \frac{1}{a}\nabla\Phi_W , \tag{3}$$

$$\frac{\partial \delta_D}{\partial t} + \frac{1}{a}\nabla\cdot(1+\delta_D)\mathbf{v}_D = 0 , \tag{4}$$

for the dark matter, and

$$\frac{\partial \mathbf{v}_B}{\partial t} + \frac{1}{a}(\mathbf{v}_B \cdot \nabla)\mathbf{v}_B + \frac{\dot{a}}{a}\mathbf{v}_B = -\frac{1}{a}\nabla\Phi_G , \tag{5}$$

$$\frac{\partial \delta_B}{\partial t} + \frac{1}{a}\nabla\cdot(1+\delta_B)\mathbf{v}_B = 0 , \tag{6}$$

for the baryons.

The functions are expressed in comoving coordinates $\mathbf{r}$ and the universal time t. Here, $\mathbf{v}_D$, p_D, and δ_D, are, respectively, the velocity, pressure, and overdensity $[\delta_D \equiv (\rho_D - \bar{\rho}_D)/\bar{\rho}_D]$ of the dark matter, with similar definitions applying to the baryons. The potential terms in Eqs. (3) and (5) contain $\nabla\Phi_G$, which is the gradient of the Newtonian gravitational potential, generated by inhomogeneity in the matter distribution: namely,

$$\nabla\Phi_G(r) = 4\pi G a^2 \int_{-\infty}^{r} (\delta_D \bar{\rho}_D + \delta_B \bar{\rho}_B) dr . \tag{7}$$

Additionally, Eq. (3) contains $\nabla\Phi_W$, which is the potential barrier of the wall, itself located at the position

$r_W(t)$. With the reasonable assumption that the walls are thin, compared to any other physical scale involved, we take this to be a hard-core potential, infinite at $r_W(t)$ and zero elsewhere.

In order to study the case of wall motion through a cold gas, we have found it most useful to divide the fluid in thin layers, each moving with a different speed, and then follow the evolution of each separately. Here, we treat the gas as collisionless, so that the various fluid layers can move through each other without interacting. Then, the pressure term p_D is zero. By changing partial derivatives to convective derivatives, i.e., moving with each fluid layer, the gradient term $(1/a)(\mathbf{v}\cdot\nabla)\mathbf{v}$ is also eliminated.

We further simplify Eq. (3) by comparing the strength of the gravitational and the damping terms. For the baryons, gravitation is the only driving force, while for the dark matter the dominant term is the wall barrier. Even the damping due to the universal expansion is larger than the effect of self-gravity of the dark matter. A comparison between $(\dot{a}/a)\mathbf{v}_D$ and $(1/a)\nabla\Phi_G$ shows that neglecting the latter term leads only to roughly 10% error in the computation of the dark matter density profile; thus, we neglect that term in Eq. (3).

Expressing the fluid equations in comoving coordinates (we define $v=av$ and $R=ar$), and changing the partial derivatives to convective derivatives, we get

$$\frac{Dv_{Di}}{Dt}+\frac{2\dot{a}}{a}v_{Di}=-\frac{1}{a^2}\frac{d\Phi_W}{dr}\ , \tag{8}$$

$$\frac{D\delta_{Di}}{Dt}+\frac{d\delta_{Di}}{dr}v_{Di}=0\ , \tag{9}$$

for the dark matter, and

$$\frac{Dv_{Bi}}{Dt}+\frac{2\dot{a}}{a}v_{Bi}=-\frac{1}{a^2}\frac{d\Phi_G}{dr}\ , \tag{10}$$

$$\frac{D\delta_{Bi}}{Dt}+\frac{d\delta_{Bi}}{dr}v_{Bi}=0\ , \tag{11}$$

for the baryons. The index i refers to the ith fluid layer, with its own comoving speed v_i and comoving density ρ_i. The comoving coordinate r, along which we want to integrate, is, of course, normal to the wall.

In Eq. (8), the only physical scale involved is the comoving distance traveled by the infinite wall barrier Φ_W. We only need to numerically integrate the dark matter equations for one value of the comoving wall speed, say $v_0=5\times10^{-3}=1500$ km/s. The choice of a different v_0 leads only to a rescaling of the size of the dark matter profile. Thus, the width of the dark matter wake is proportional to the product v_0t. We have found that the baryon wake grows in a similar manner to the dark matter profile. We will outline our results in Sec. V.

IV. OUTLINE OF THE NUMERICAL CODE

The numerical code studies the phase-space evolution of each gas layer [see Eqs. (8)–(11)] separately. We have chosen an array of 1200 layers for both dark matter and baryons. The position of these layers are evolved in time, and from the relative distance of the center of the layers we can determine the evolution of the relative density as well. The evaluation of the gravitational force follows directly from using the density profiles and a discretized version of Eq. (7).

In order to have the optimal spatial resolution at all times of the integration and because of the scaling behavior we expected for the density profiles, we have found it useful to perform the simulation in logarithmic spatial coordinates, with a dynamic range of three decades on both sides of the origin. The initial thickness of the layers is chosen to be constant in logarithmic units. Additionally, we have kept the origin of coordinates at the position of the wall throughout the simulation and we have used periodic boundary conditions. The time step is chosen so that the variation of the scale factor is $\Delta a/a=0.1\%$ at each time step.

We can easily discretize the equation for the ith layer of the dark matter starting from $\Delta v_{Di}/\Delta t=-2[\Delta a/(a\Delta t)]v_{Di}$, valid for all layers which are not bounced by the wall within the time step Δt. The equation can be rewritten as

$$\Delta v_{Di}=-2(\Delta a/a)v_{Di}\ , \tag{12}$$

with the additional condition, from momentum exchange, that $\Delta v_{Di}=2(v-v_{Di})$ when the ith layer bounces off the wall. To calculate the density ρ_i, given its position r_i, we note that $\rho_i\propto(r_{i+1}-r_{i-1})^{-1}$, for both dark matter and baryons. A similar procedure to discretization leads to the following equation of motion for the baryons:

$$\Delta v_{Bi}+\frac{\Delta a}{a}v_{Bi}=-\frac{1}{t}\frac{\Delta a}{a}\int_{-\infty}^{r_i}(\delta_D\Omega_D+\delta_B\Omega_B)dr\ , \tag{13}$$

with $\Omega_{D,B}\equiv\bar{\rho}_{D,B}/\bar{\rho}_{\rm tot}$ and where the integral is a summation over all layers behind the ith baryon layer under consideration. Incidentally, in rewriting Eq. (10) we used $4\pi G\bar{\rho}_{\rm tot}t_0^2=\frac{2}{3}$.

We now briefly discuss the modification to the value of the constant K in Eq. (2), due to the multiple collision effect we mentioned in Sec. II. We performed a preliminary run choosing a constant wall speed v, and we studied the formation of the dark matter wake. The particles are reflected and gain momentum, only to lose it gradually due to the universal expansion. Eventually they are reflected again, but their speed relative to the wall at the second collision is smaller. Collision after collision, the particle layers gain less momentum and their speed tends to that of the wall. The density profile reaches a scaling regime, as already predicted in Ref. [17]. The value of K rapidly converges to $K\simeq2.7$, as the wake reaches the scaling regime. From Eq. (2), it is therefore clear that the modification to the wall speed due to this effect is small, namely, $\delta v/v\simeq15\%$.

As a check of the numerical code, we compared the thickness of the dark matter wake to the analytical estimate we had presented in Ref. [17]. The wake is one-sixth of the void left behind, in good agreement with the previous estimate. The scaling behavior of the dark matter density profile is also a good indicator of the validity of our code.

V. RESULTS

We operate in the matter-dominated regime, namely $a \propto t^{2/3}$. With $\Omega_0 \equiv \tilde{\rho}_{\rm tot}/\tilde{\rho}_{\rm crit}$, this is strictly true only when $\Omega_0 = 1$. However, this expansion law effectively holds whenever the scale factor $a < \Omega_0$, certainly $z > 5$, the epoch we are studying. For our simulation, we fix the dark matter density at 90% of the total energy density of the Universe, with the remaining 10% in baryons. Thus, $\Omega_D = 0.9$ and $\Omega_B = 0.1$.

The simulation confirms the analytical results for the evolution of the dark matter profile. The width of the wake is roughly one-sixth the width of the void behind the domain wall, and the relative overdensity profile of the dark matter scales with $v_0 t$, namely, $\delta_D = \delta_D(r/v_0 t)$, in comoving coordinates. The density profile of the dark matter in the scaling regime is shown in Fig. 1. The distance axis units have been rescaled to show what the dark matter profile looks like at the present epoch, for a typical domain wall speed of 1500 km/s.

The baryons are tugged by the gravitational potential gradient of the dark matter wake and their own self-gravity. The baryons form a "tug" wake behind the wall, in the region where the dark matter has been swept away. The baryon density profile also reaches a scaling solution $\delta_B = \delta_B(r/v_0 t)$, but only after a time $t_c \simeq 15 t_i$, when the peak overdensity saturates at a value of approximately $\delta_B^{\rm max} \simeq 0.4$. Such a baryon profile is shown in Fig. 2. We find that this maximum overdensity depends only on the ratio between the mean densities of the dark matter and the baryons. It is a maximum in the case we have presented, namely, in a Universe that is 90% dark matter dominated.

We find that the width d_B of the baryon wake is about one-half of the dark matter void behind the domain wall,

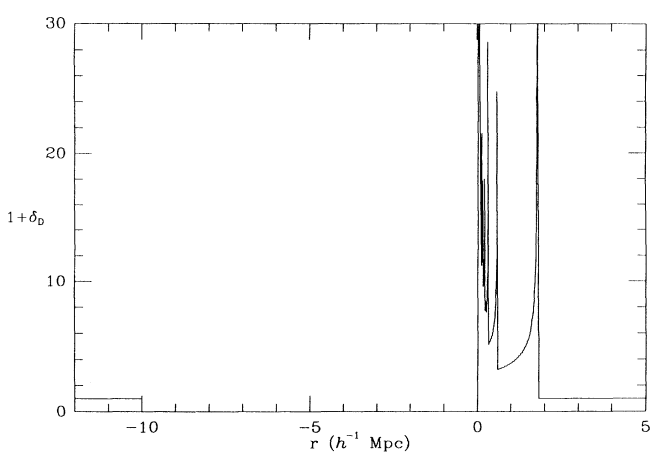

FIG. 1. Dark matter density profile, in units of the average dark matter density. The abscissa is the comoving distance (in h^{-1} Mpc) from the wall, which is located at the origin. The positive axis is in the direction of motion of the wall. The density profile evolves self-similarly with time. The edge of the void region lies at $|r| = 10h^{-1}(v_0/5\times10^{-3}c)(t/t_0)$ Mpc, where v_0 is the comoving wall speed and t_0 is the present age of the Universe. We show the case $v_0 = 5\times10^{-3}c$ and $t = t_0$.

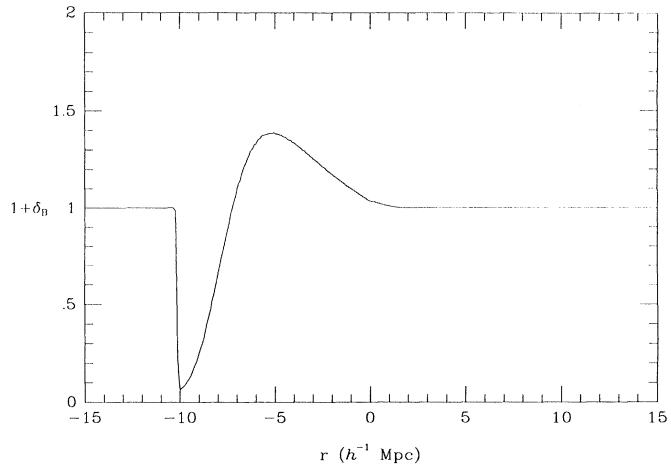

FIG. 2. Baryon density profile, in units of the average baryon comoving density, with the same units as Fig. 1.

namely, $d_B \simeq 0.5 v_0 t$. The crest of the baryon wake moves at a constant comoving speed of $0.5 v_0$. For a typical domain wall moving at speed $v_0 = 1500$ km/s, the present thickness of the dark matter void is $v_0 t_0 = 10h^{-1}(v_0/5\times10^{-3})$ Mpc, and $d_B \simeq 5h^{-1}(v_0/5\times10^{-3})$ Mpc, with the crest moving at $750(v_0/5\times10^{-3})$ km/s. Thus, this domain wall scenario leads to cosmologically very interesting peculiar velocities in the baryon wake. In the next section we will discuss this and other implications of the model.

VI. DISCUSSION

We have found that, in this overdamped regime, the domain wall scenario leads to the formation of large structures in both the dark matter and the baryons. This process of structure formation occurs relatively quickly, reaching a scaling solution at a redshift $z_{\rm sc} = a_{\rm sc}^{-1}$ when the scale factor grows tenfold. Thus, since $a_{\rm in}$ can be as small as 10^{-3}, $a_{\rm sc}$ can be as small as 10^{-2}; hence, structure formation can occur as early as redshifts $z_{\rm sc} \sim 100$. The voids and wakes trace the domain wall network as a whole, which is "frozen in" in comoving coordinates because of the high friction with the dark matter. The thickness of these structures grows linearly with time; however, the total thickness of the wakes and voids, of order a few Mpc today, is small compared to the network scale of order $50h^{-1}$ Mpc. The same picture holds for the baryons. They also trace the domain wall network by trailing behind the dark matter wakes. The baryons accumulate in sheetlike structure that are separated by distances corresponding to the wall network scale. An interesting consequence of this model is that sheets of baryon overdensity have a typical thickness of $5h^{-1}$ Mpc, if we *independently* fix the baryon overdensity peak to be moving with a velocity close to 750 km/s (by choosing a typical $v_0 \simeq 1500$ km/s).

This model can thus accommodate recent observational data, showing large peculiar velocities of order several hundreds of km/s in galaxies, which appear to be grouped in sheetlike structures of the aforementioned

scales. Note that the dark matter and the baryon overdensities are largely segregated. It may thus be possible to account for much of the controversy in various estimates of Ω_0, which at smaller scale appears to be $\Omega_0 \simeq 0.1-0.2$, and $\Omega_0 \simeq 0.5-1$ on the larger scales.

Despite the early formation of sheets and voids in the dark matter, the resulting perturbation in $\Delta T/T$ of the CMBR due to gravitational effects are small. These are of the order $\Delta T/T \sim 10^{-6}$ on angular scales of a few degrees. We have not computed the effects of reionization and reheating, i.e., the y parameter, because of our incomplete knowledge of galaxy formation within the sheets. We now address this last remaining issue.

To study the formation of structure within the sheets requires a knowledge of the three-dimensional topology of the domain wall network. It is still true that the basic effects of wake and void formation are described by our one-dimensional computation, mostly because these involve accumulation of matter in a direction perpendicular to the surface of the walls. However, we have found that the perturbations saturate in one dimension, and we need to consider the effects of curvature of the domain wall network on the baryon perturbations.

Because the domain wall network is irregular, different sections of the walls move at different speeds. We have found that both the dark matter and the baryon density profiles tend to scaling solutions, with a scale directly proportional to the wall speed. Thus, the total mass accumulated in front and emptied behind the wall is directly proportional to the local wall speed, which, in turn, is determined by the local value of the domain wall curvature. Thus, there is a *direct* relation between perturbations in matter and the topology of the wall network. The largest accumulation of mass occurs near sections of the network that are highly curved and thus move more quickly. Conversely, wall sections that are flat disturb the surrounding dark matter very little.

The curvature effects lead to peculiar velocity flows tangential to the wakes, and outside of the wakes as well. We can estimate the typical peculiar velocities v induced in the surrounding medium by the wake network in the following way. The gravitational field will be of the same order as that of a typical dipole with moment $p = M\bar{d}$, mass $M \sim \rho_D \bar{d}\bar{R}^2$, and an average coherence scale $\bar{R}$. Thus $\dot{v} \sim 4\pi G p/\bar{R}^3 = 4\pi G \rho_D (\bar{d}/\bar{R})$, which yields $v \sim v_\perp (\bar{d}/\bar{R})$. Here $\bar{d} \sim v_\perp t$ and $v_\perp$ is the speed of the baryons within the wakes in a direction perpendicular to the network surface. As one may expect, when the "dipole size" $\bar{d}$ is comparable with the "dipole separation" $\bar{R}$, the component of the velocity tangential to the wakes is similar to $v_\perp$. For the typical values we used in the paper $\bar{d}/\bar{R} \sim \frac{1}{5}$, we get $v \sim 100$ km/s. Since the coherence length of this velocity field is $\sim \bar{R} \gg vt$, we see that structure formation by the effects we just calculated is not very significant.

While the "high barrier" domain wall model cannot, by itself, generate baryon collapse and galaxy formation, it may be able to accelerate the formation of the large-scale structure once the small-scale collapse has already started due to some other mechanism. In this sense, this domain wall model can be complementary to the standard CDM model. In fact, because of the time dependence of the wake thickness, most of the dark matter is swept by the walls at $z \simeq 0-2$, just during the period when the standard CDM model predicts that most of the collapse and merging processes take place. It is hard to evaluate what changes the wake formation may imply for the standard scenario, without more detailed numerical work. This phase of the collapse is poorly known already in the standard model. The number of bright galaxies and the efficiency of the star formation may be greatly affected by the violent sweeping action of the walls, and this may, in turn, result in an enhancement of the galaxy correlation function up to scales close to that of the wall network. A more detailed simulation of the nonlinear phase of collapse may help in determining if this enhancement actually occurs. Clearly, this domain wall model is very interesting from the point of view of structure formation, and should be investigated further by more detailed three-dimensional simulations.

ACKNOWLEDGMENTS

We wish to thank P. Coppi, B. A. Gradwohl, and A. Chakravorty for useful discussions and comments. A. M. was supported in part by the DOE at The University of Chicago, and by NASA through Grant No. NAGW-2381 at Fermilab. J.M.Q. was supported in part at The University of Chicago by NSF through Grant No. AST90-22629 and the Robert R. McCormick Foundation.

[1] G. Efstathiou, N. Kaiser, W. Saunders, A. Lawrence, M. Rowan-Robinson, R. S. Ellis, and C. S. Frenk, Mon. Not. R. Astron. Soc. **247**, 10 (1990).
[2] T. J. Broadhurst, R. S. Ellis, D. C. Koo, and A. S. Szalay, Nature **343**, 726 (1990).
[3] M. Davis, G. Efstathiou, C. S. Frenk, and S. D. M. White, Astrophys. J. **292**, 371 (1985).
[4] M. Devlin *et al.*, in Results from the Third Flight of the Millimeter Anisotropy Experiment, Proceedings of the NAS Colloquium on Physical Cosmology, Irvine, California, 1992 (unpublished).
[5] K. M. Gorski, Astrophys. J. **398**, L5 (1992).
[6] A. Vilenkin, Phys. Rep. **121**, 263 (1985).
[7] N. Turok, Phys. Scr. **T36**, 135 (1991).
[8] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Reading, MA, 1990), Chaps. 7 and 8.
[9] C. Hill, D. Schramm, and J. Fry, Comments Nucl. Part. Phys. **19**, 25 (1989).
[10] L. Kawano, Phys. Rev. D **41**, 1013 (1990).
[11] B. S. Ryden, W. H. Press, and D. N. Spergel, Astrophys. J. **347**, 590 (1989).
[12] T. W. B. Kibble, J. Phys. A **9**, 1387 (1976); A. E. Everett, Phys. Rev. D **10**, 3161 (1974).
[13] M. C. Huang and P. Sikivie, Phys. Rev. D **32**, 1560 (1985).
[14] A. Chakravorty and A. Massarotti, Fermilab Report No. 91/306-A (unpublished).

[15] S. M. Allen and J. W. Cahn, Acta Metall. **27**, 1085 (1979).
[16] V. de Lapparent, M. J. Geller, and J. Huchra, Astrophys. J. **302**, L1 (1986); **332**, 44 (1988).
[17] A. Massarotti, Phys. Rev. D **43**, 346 (1991).
[18] A. Massarotti, Fermilab Report No. 91/104-A (unpublished).
[19] J. Peebles, *The Large Scale Structure of the Universe* (Princeton University Press, Princeton, NJ, 1980), Chap. 2.