Evolution of light domain walls interacting with dark matter

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In this paper we discuss the evolution of domain walls generated in the early Universe considering, unlike the previous studies, an interaction between the walls and a major gaseous component of the dark matter. It is assumed that the walls can reflect the particles elastically and with a reflection coefficient of unity. We discuss a toy Lagrangian that could give rise to such a phenomenon. In the simple model studied we obtain highly nonrelativistic and slowly varying speeds for the domain walls $[\sim 10^{-2}(1+z)^{-1}]$ and negligible distortions of the microwave background. In addition, these topological defects may provide a mechanism of forming the large-scale structure of the Universe, by creating fluctuations in the dark matter $\delta\rho/\rho \sim 1$ on a scale comparable to the distance the walls move from the formation (in our model this distance could be even tens of Mpc). The characteristic scale of the wall separation can be easily chosen to be of the order of 100 Mpc instead of being restricted to the horizon scale, as usually obtained.

I. INTRODUCTION

The cosmological consequences of primordial phase transitions associated with scalar fields have been the subject of many studies in recent years. The topological defects created in the transitions, such as domain walls, strings, and monopoles, are potentially of great interest for cosmology, since they could supply seeds for the formation of the large-scale structure of the Universe. Specifically, domain walls are sheetlike regions of a false vacuum in between domains having different and disconnected vacuum ground states of the scalar field. The simplest and most studied model involves a real scalar field with a quartic potential and a negative sign for the mass term. After the phase transition the field rolls down to one of the two zero-temperature minima for the potential. This leaves a domain structure on scales bigger than the correlation length of the field, resembling closely what happens in an Ising model.[1] When originally introduced, the phase transitions considered were on the grand-unified-theory (GUT) scale.[2] The trouble is that domain walls on the GUT scale rapidly become the dominant form of matter in the Universe and produce overly big distortions in the present microwave background.

Recently, interest in domain walls has been raised again considering late phase transitions (at $z \sim 100$) that would generate so-called "soft" domain walls.[3] These walls may never be massive enough to distort the microwave background, but they may, *a priori,* be a dominant gravitational component of the present Universe, triggering the formation of galaxies and changing the expansion rate. These possibilities have been excluded by a numerical study[4] of the evolution of the field itself through the phase transition and afterwards, as the walls appear and evolve by their surface tension. The domain walls soon reach relativistic speeds, and the average scale of the system becomes comparable to the horizon scale, making these walls unusable for the formation of the large-scale structures we see.[5,6] Very similar results have been obtained[7] by considering directly the evolution of the walls after the phase transition. In that calculation the approximation taken is that the wall thickness is much smaller than the radius of curvature of the wall surface.

The problems mentioned arise because of the lack of energy dissipation in the models considered; the mass energy stored in the walls gets efficiently converted into their kinetic energy, rapidly raising them to relativistic speeds. We therefore consider the effect of introducing in the equation of motion of the walls a friction term that is a function of the wall speed relative to the background matter and its density. The idea of studying the consequences of friction on domain walls can be traced back to Refs. 1, 8, and 9, but it was never fully developed because it was introduced in the context of GUT scale phase transitions, in which case including friction would even worsen the problems pointed out previously. In this paper we will consider much-lower-energy scales, of the same order as those obtained in Ref. 3. It will be shown that indeed there exists an interesting range of the wall energy density for which the average "interwall" distance is of the order of 100 Mpc today, and that these domain walls are compatible with the limits on the anisotropy of the microwave background.

The paper is organized in the following way. In Sec. II we derive the equation of motion of an element of a domain wall without any friction term other than the usual one due to the universal expansion; in Sec. III we concentrate our attention on the friction pressure arising when walls move through a homogeneous gas reflecting all incident particles elastically; in Sec. IV we introduce the results of Sec. III into the equation of motion previously calculated; in Sec. V we discuss what kind of particle Lagrangian may lead to the premises of this paper and the consequences of our model on the microwave background.

Throughout the paper we will assume, for simplicity, that the Universe is at critical density. Most of our results are independent of this assumption.

II. EQUATION OF MOTION IN THE ABSENCE OF FRICTION

In order to approach the problem, we will assume, first of all, that we are dealing with domain walls late enough after the phase transition so that the thin-wall approximation can be considered roughly valid.[7] We are therefore interested in the motion of sharp interfaces under their surface tension. The shape of the network, which is related to the details of the model chosen, will turn out to be unimportant in the discussion. The important assumption made here is that we are dealing with a class of models with two or more degenerate values for the vacuum ground state, so that the driving force of the motion of the walls is only their surface tension. Throughout the following calculations we will assume that these kinks are moving nonrelativistically. This will turn out to be a sensible choice in the framework of the scenario we present in this paper (see Sec. IV).

Many approaches may be considered to get the local equation of motion of the walls; the most direct of these is just to start with the well-known equation of motion for real scalar fields.[4,10]

$$\ddot{\Phi}+3\frac{\dot{a}}{a}\dot{\Phi}-\frac{1}{a^2}\nabla^2\Phi=-\frac{\partial V}{\partial\Phi}\ , \tag{1}$$

where a is the scale factor, given by $a=t^{2/3}$ if $\Omega=1$ (we express t in $2H_0^{-1}/3$ units, H_0 being the present value of Hubble constant; $a=1$ today). Equation (1) is expressed in comoving coordinates and universal time. After the phase transition there are regions of different vacua separated by kinks [which are classical solutions of Eq. (1)].

In general, we can define the two-dimensional (2D) space on which $\partial V/\partial\Phi=0$ as the surface of a kink. At each point of the surface, we can choose a coordinate system such that x is the normal axis. If the curvature of the surface is much smaller than the inverse of the wall thickness Δ^{-1}, we can locally characterize the kink by a function $\Phi(a[x-r(t)])$ of the x coordinate only, where $r(t)$ describes the motion of the kink surface along the normal axis. The function Φ can be inserted in Eq. (1) to find the differential equation obeyed by $r(t)$ at any given surface point. The calculation is easily performed when we recall that

$$\nabla^2=\frac{\partial^2}{\partial x^2}+(\nabla\cdot\hat{\mathbf{x}})\frac{\partial}{\partial x}\ , \tag{2}$$

where $\hat{\mathbf{x}}$ is the unit vector perpendicular to the kink surface. The divergence $\nabla\cdot\hat{\mathbf{x}}$ is the curvature scalar at the point considered, and we can define[11]

$$\nabla\cdot\hat{\mathbf{x}}\equiv 2/\tilde{r}\ . \tag{3}$$

In this way from Eq. (1) we get

$$\left[-\ddot{r}\Phi'+a\dot{r}^2\Phi''-2\frac{\dot{a}}{a}\dot{r}\Phi'\right]-3\frac{\dot{a}}{a}\dot{r}\Phi' -\frac{1}{a^2}\left[a\Phi''+\frac{2}{\tilde{r}}\Phi'\right]=-\frac{1}{a}\frac{\partial V}{\partial\Phi}\ , \tag{4}$$

where Φ' and Φ'' are derivatives with respect to the whole argument of Φ.

Evaluating Eq. (2) at $x=r(t)$ (i.e., at the kink surface), we see that Φ' becomes very big when $\Delta\to 0$ ($\Phi'\sim\Delta^{-1}$), while the Φ'' term is very small (it would be exactly $\Phi''=0$ if the wall were straight). In the thin-wall approximation we therefore get to the final expression

$$\ddot{r}+5\frac{\dot{a}}{a}\dot{r}=-\frac{1}{a^2}\frac{2}{\tilde{r}}\ , \tag{5}$$

where $\tilde{r}/2$ is the inverse of the curvature scalar at the chosen point of the wall network.

This expression can be rewritten in physical coordinates, defining $\tilde{R}\equiv a\tilde{r}$ and $v\equiv a\dot{r}$, where v is the peculiar velocity of the walls with respect to the comoving frame. We therefore finally get to

$$\dot{v}+4\frac{\dot{a}}{a}v=-\frac{1}{a^2}\frac{2}{\tilde{R}}\ . \tag{6}$$

If there were no universal expansion (set $a=1$ constant), then Eq. (4) would be simply $\dot{v}=-2/\tilde{R}$. Defining σ as the mass-energy density of the walls, then $P_T=2\sigma/\tilde{R}$ would be the pressure due to the surface tension, exactly the same form that one obtains in condensed matter. This also reminds us that Eq. (4) is just Newton's second law divided by σ.

III. FRICTION TERM

We now calculate the pressure exerted on the walls moving with speed $v\ll 1$ through some homogeneous medium interacting with it. We are going to study only the case in which the medium remains homogeneous throughout all the period of evolution considered. This can be considered valid, for example, if perfectly reflecting walls move so little that they are not able to reshuffle the bulk of the matter, i.e., if they move a small fraction of the distance between each other, so that no particle interacts with two different walls in a cosmological time (see also the discussion in Sec. IV). In all the following we are restricting ourself to this simple case.

We begin by writing down the general expression for the friction force acting on the domain walls as they move nonrelativistically through a homogeneous gas. In the following derivation we will assume that each particle couples only with the walls. We will also assume that the walls reflect elastically all the incident particles, regardless of their energy at the impact. This condition could be relaxed, as we will discuss at the end of this section.

For a nonrelativistic gas we can write that the pressure exerted by the gas on the wall is given by (see Appendix A)

$$P_f = -2mn \int_y^{\infty} B^{-2\alpha}(y-y_x)^2 f(|y_x|)dy_x + 2mn \int_{-\infty}^{y} B^{-2\alpha}(y-y_x)^2 f(|y_x|)dy_x \; , \tag{7}$$

where $B \equiv m/T$, $y = B^{\alpha}v$, and $y_x = B^{\alpha}v_x$.

Let us consider the limits in which $y \ll 1$ and $y \gg 1$. In the former case the thermal speed of the particles is much greater than the speed of the domain wall, since the average thermal momentum of the particles is $\bar{p} \sim T$; in the latter case the wall is moving through particles effectively at rest and the volume spanned remains depleted of the gas. The case $y \gg 1$ will turn out to be the most interesting in our present discussion.

For $y \ll 1$ changing the variables inside the integrals ($y_1 = y - y_x$) and expanding $f(|y_x|)$ in power series around y_1, we get

$$P_f = -4mnB^{-2\alpha} y \int_0^{\infty} y_1^2 f'(y_1)dy_1 = 4mn(T/m)^{\alpha} vF \; , \tag{8}$$

where $F \equiv -\int_0^{\infty} y_1^2 f'(y_1)dy_1$ is a constant of order unity. For $y \gg 1$ instead we obtain

$$P_f = 2mnv^2 \; , \tag{9}$$

since $f(y_x) \sim 0$ for $y_x >> 1$, and therefore we can substitute $(y-y_x)^2$ by y^2 in the integrals. Such a result is not surprising if we can recall that in this case the momentum exchange per particle is $\Delta p = 2mv$ and that the number of particles hitting the unit area per unit time is nv.

The case in which the gas is relativistic is even easier, since the number of particles hitting the wall per unit time is simply given by n ($c=1$) on both sides of the surface. Taking $v \ll 1$, we get

$$P_f = n \left[\int_0^{\infty} p_x(1+v) f(|p_x|)dp_x - \int_0^{\infty} p_x(1-v) f(|p_x|)dp_x \right] = 2nv\overline{p_x} \; , \tag{10}$$

so that $P_f \sim vT^4$, which is the limit discussed in the review.[1]

We can also give an evaluation of the average thermal speed of the particles interacting with the walls, e.g., for light neutrinos and for gas following the Boltzmann distribution, supposing that the particles decouple at a certain z_d (note that throughout the paper, $z+1 = a^{-1}$). The momentum of the particles shifts with the expansion of the Universe so that $\bar{p}_d/\gamma m\bar{v}(z) = (z_d+1)/(z+1)$, where $\gamma^{-1} = (1-\bar{v}^2)^{1/2}$, if at decoupling $T_d \gg m$, then $\bar{p}_d \sim T_d$, if $T_d \ll m$, then $m\bar{v}_d^2 \sim T_d$, so that $\bar{p}_d \sim (mT)^{1/2}$. Assuming the particles to be nonrelativistic today, we get

$$\bar{v}(z) \sim \left[\frac{T_d}{m} \right]^{1/2} \frac{z+1}{z_d+1}, \quad T_d \ll m \tag{11}$$

and

$$\bar{v}(z) \sim \left[\frac{T_d}{m} \right] \frac{z+1}{z_d+1}, \quad T_d \gg m \; . \tag{12}$$

If we assume neutrinos of mass $m \sim 10$ eV and $T_d \sim 1$ MeV, we get $\bar{v}(z) \sim 10^{-5}$ at $z=0$. This result will turn out to be useful in the following discussion.

In closing this section we return briefly to our initial assumptions. Although we are interested here in studying the consequences of a reflection coefficient close to unity, there could be cases in which one has to deal with an energy-dependent partial transmission of the incident particles through the walls. This would lead to a class of solutions in which the walls may decouple from the matter after a certain stage, when their speed with respect to the matter becomes bigger than a critical value. These possibilities are at present under investigation and go beyond the goals of this paper.

IV. DOMAIN WALLS AND FRICTION: A SIMPLE CASE

The main conclusion of Sec. III is that, if the matter interacting with the walls is nonrelativistic, there are two different regimes for the friction pressure P_f, depending on the value of $y \equiv (m/T)^{\alpha} v \sim v/v_T$ (where v_T is the average thermal speed of the particles). Certainly, one can consider a particle mass large enough so that the evolution of the network takes place in the $y \gg 1$ regime. In fact, we will see shortly that this statement is valid for any reasonable dark-matter candidate, including light neutrinos.

We therefore start our analysis by introducing the friction term for $y \gg 1$ [Eq. (7)] into the equation of motion [Eq. (4)].[12] We define $\rho_m \equiv mn$ so that $\rho_m = \rho_{m0}/a^3$, where ρ_{m0} is the mass density of the matter interacting with the walls today. The equation of motion in the presence of the friction is

$$\dot{v} + 4\frac{\dot{a}}{a}v + \frac{v^2}{Ka^3} = -\frac{1}{a^2}\frac{2}{\tilde{R}} \; , \tag{13}$$

where $K = \sigma/2\rho_{m0}$ and the constant σ is the energy density of the walls. If the friction dominates the motion, then $\dot{v} \ll v^2/Ka^3$ and $2\dot{a}/a \ll v/Ka^3$, and we can neglect these terms. At the end of the section, we will show that, given the cosmological parameters characterizing the model, these conditions are always satisfied. We therefore remain with the important result

$$v^2 = -\frac{2Ka^3}{\tilde{R}} \; . \tag{14}$$

Equation (12) is valid as long as the medium is homogeneous up to the passage of the kinks and the speed of the walls is far bigger than the thermal motion of the gas.

One of the assumptions of the model here presented is that, throughout their evolution, the walls move little compared to interwall distance. This ensures that domain walls always sweep through a homogeneous and thermalized gas, a condition at the basis of the calculations of Sec. III. We can rephrase this requirement by saying that, up to corrections, the network conformally stretches with the universal expansion.

However, it is very important to determine the small average displacement $\bar{d}$ of the network from the very initial configuration (due to the peculiar motion of the kinks), since the volume swept by the domain walls remains depleted of dark matter. This depletion represents a very strong density fluctuation on scales possibly of the order of Mpc or tens of Mpc, and could trigger the formation of galactic structures in close connection with the presence of the walls.

For our present analysis it is sufficient to deal with the average characteristics of the system. We therefore average the quantities present in Eq. (12) over the network surface S contained in a volume V much greater than its typical radius of curvature. Defining $\overline{R}$ as a point-by-point average of the distance of two neighboring walls integrated over the surface S, we get

$$\frac{1}{\beta\overline{R}}=\overline{\left[\frac{2}{R}\right]}\equiv\frac{1}{S}\int\frac{2}{R}dS\ , \tag{15}$$

where β is a constant close to unity,[13] so that we finally write

$$\overline{v}\equiv(\overline{v^2})^{1/2}=\left[\frac{Ka^3}{\beta\overline{R}}\right]^{1/2}\sim\left[\frac{K}{\beta\overline{R}_0}\right]^{1/2}a\ , \tag{16}$$

where $K=\sigma/2\rho_{m0}$ and $\overline{R}_0\sim\overline{R}/a$, with $\overline{R}_0$ being the average interwall distance today. It is interesting to note that the average comoving speed $\overline{v}/a$ remains roughly a constant throughout the evolution of the network.

Our goal would be to determine σ given $\overline{R}_0$ and $\overline{d}$. Before doing that we should slightly modify Eq. (14), taking into account the following correction. The friction term we utilize in Eq. (11) is based on the assumption that only one reflection occurs to each particle, but actually the particles interact several times with the same wall. This is just a consequence of the kinematic rules of the expanding Universe. We give the details of this correction in Appendix B. The result expressed in Eq. (14) does not change in form, but we have to substitute K by $K/6$, so that we get to the final result

$$\overline{v}\sim\left[\frac{\sigma}{12\beta\rho_{m0}\overline{R}_0}\right]^{1/2}a\ . \tag{17}$$

To proceed we now need an estimate of K. If we associate the walls with the peaks of the distribution of galaxies observed in the survey,[6] which suggests that the domain walls may be related to the very-large-scale clustering process, then the scale of our network today is $\overline{R_0}\sim120h^{-1}$ Mpc. Since the average distance traveled by the walls of $\overline{d}\equiv\overline{v}$ (the present age of the Universe is $t_0=1$ if $\Omega=1$), we obtain $K/6=\sigma/12\rho_{m0}=6\beta10^{-2}\overline{d}^2$; this can also be written as

$$\frac{\Omega_{w0}}{\Omega_{m0}}\sim12\beta\overline{d}^2\ , \tag{18}$$

with $\Omega_{w0}/\Omega_{m0}\sim\sigma/\rho_0\overline{R}_0$ [or $\sigma\sim1.2h\beta(\overline{d}/20h^{-1}$ Mpc$)^2(\overline{R_0}/120h^{-1}$ Mpc)MeV^3] by geometry. Assuming $\Omega_{m0}=1$, this would yield $\Omega_{w0}\sim1.2\beta10^{-3}(\overline{d}/20h^{-1}$ Mpc$)^2$. Equation (16) shows that in a friction-dominated model the domain walls never get to dominate the energy density of the Universe. Note that the fraction of dark matter swept by the domain walls during their evolution is only $\sim\overline{d}/\overline{R_0}$.

We can easily derive

$$\overline{v}\sim10^{-2}\left[\frac{\overline{d}}{20h^{-1}\text{Mpc}}\right]a\ , \tag{19}$$

which says that our initial assumption $v\ll1$ is satisfied by a big margin.

At this point one could note that the condition $y\equiv mv/T\gg1$, while amply satisfied today, may not be valid as well during all the past evolution of the network. To see this just take, for example, the estimate of the average particle speed in the case of light ($m\sim10$ eV) neutrinos, made in Sec. III. The wall speed, as expressed in Eq. (17), is valid independently of the particle mass, as long as $y\gg1$. Comparing Eq. (17) with Eq.(10), one can easily see that $y<1$ at $z>30$, for $m\leq10$ eV neutrinos. The heavier the particles are, the higher this limiting z value. When $y<1$ one can introduce the friction term given by Eq.(5) ($P_f\sim vT^4$) in the equation of motion and then follow the same arguments used at the beginning of this section. This correction to the first phase of the expansion would anyway leave unaltered the discussion of Eqs. (12)–(17), since most of the evolution of the network takes place at $z<5$. We therefore see the validity of our choice of the friction term made earlier.

Let us consider our self-consistency check, going back to our original Eq. (11). We know that in a universe with critical density, $\Omega_{m0}=1$ and $a=t^{2/3}$, so that $\dot{v}/v=2/3t$ at each point of the network; we therefore get

$$\dot{v}=\frac{2}{3}\frac{v}{t}\ll12\left[\frac{\rho_{m0}\overline{R}_0}{\sigma}\right]\frac{v^2}{\overline{R}_0t^2}$$
$$=\frac{10^4}{6\beta10^{-2}}\frac{v^2}{t^2}\rightarrow v\gg4\times10^{-6}\beta t\ , \tag{20}$$

and

$$4\frac{\dot{a}}{a}v\ll\frac{1.5\times10^5v^2}{\beta t^2}\rightarrow v\gg1.6\times10^{-5}\beta t\ . \tag{21}$$

As previously anticipated, at any time considered we meet the conditions for friction-dominated motion.

V. DISCUSSION

We have shown that, if domain walls formed during some late phase transition can reflect perfectly the particles of gas of a component of the dark matter, then the domain-wall network is bound to expand with the scale factor, provided that $\Omega_{w0}/\Omega_{m0}\leq1.2\beta10^{-3}(\overline{d}/20h^{-1})^2$. The coupling between the scalar field Φ and the particles in question (called the associated field Ψ) may assume the very simple form of a mass term dependent on the spatial coordinates. For the sake of discussion, take Ψ to be fermions. A toy Lagrangian for the field associated with the Ψ particle could be written as $\mathcal{L}(\psi)=\overline{\Psi}\partial\!\!\!/\Psi+[m+f(\Phi)]\overline{\Psi}\Psi$ [a Lagrangian of this form is obtained, for example, in Ref. 3, where $f(\Phi)$ is a real function of the field Φ that gives the domain structure and is assumed to get higher values within the kinks rather than outside]. Because of the presence of the kinks, the mass of the particles changes when they get close to the soliton. In the nonrelativistic case this situation is equivalent to obtaining a Schrödinger equation for free particles with a potential $V=f(\Phi(\mathbf{R}))$. We can say that $V(\mathbf{R})$ is a perfect barrier if the reflection coefficient on both sides of the kink is unity.

Now we turn to consider a possible estimate of the wall thickness, which has been, up to this point, a free parameter. If the domain walls maintain their position from the formation (in the comoving coordinate system), we are actually bound to consider second-order phase transitions taking place at a z_f not bigger than $z_f=\overline{R_0}/R_H(z_f)=\overline{R_0}/3t_f=2\times10^{-2}z_f^{3/2}\rightarrow z_f\leq 2500$ (R_H is the horizon scale at z_f) due to simple causality considerations. As a consequence, there is a lower bound to the thickness Δ of the domain walls; since the interwall distance is $\overline{R_f}\sim\Delta\sim\overline{R_0}z_f^{-1}$ at formation, $\Delta\geq 3.5\times10^{-5}$ in our units, which is $\Delta\geq 7\times10^{-2}h^{-1}$ Mpc.[14] Such a distance is far greater than the wavelength usually associated to any dark-matter-particle candidate (even for neutrinos $\lambda_{\text{thermal}}<10^4$ eV^{-1} at any z). We infer that $V(r)$ can be considered a classical barrier of height $E_{\max}$ such that for $E<E_{\max}$ the reflection coefficient is unity and for $E>E_{\max}$ it is zero. In this paper we only consider the case $E_{\max}\rightarrow\infty$.

A couple of issues still remain to be solved. The walls carry a gravitational field that shifts the frequency of the microwave-background radiation of a slight amount when this passes through the potential. Such a problem has been treated in Refs. 3 and 15.

The infinitesimal shift of the average photon energy T while the photon is moving for a time dt through the gravitational influence of a wall is given by $dT\sim\delta T+T\delta V$, where V is the gravitational potential of the wall. V is roughly given by $V\sim G\sigma R$ at a distance R from the kink surface, within a cutoff value $\sim\overline{R}/2$; $\overline{R}$ is the average interwall distance in physical coordinates at the time considered.[16] The value of V varies in time due to the evolution of the network, so that $\delta V=(\partial V/\partial t)\delta t+\nabla V\cdot\delta\mathbf{R}$ (where $|\delta\mathbf{R}|=\delta t$). We want to calculate the total shift in the temperature of the photons as they pass through the gravitational potential of a single wall, i.e., within the cutoff distance of V. If we take roughly $\partial V/\partial t\sim G\sigma\dot{\overline{R}}$ in a region of order $\overline{R}$ in size (this is clearly an overestimate), when we integrate the above expression for dT to find the total shift of the temperature, we get a term $\delta T/T=\alpha G\sigma\overline{R}^2/t$ in addition to the usual term due to the expansion; α is a fudge factor of order unity, and t is the age of the Universe at the epoch considered. The biggest distortion can be reached at the present epoch: $\delta T/T\sim10^{-8}$.

Another effect may be considered. The fluctuation in the matter density due to the sweeping action of the wall gives rise to a gravitational influence limited to the region of thickness $\overline{d}$ in which $\delta\rho_m/\rho_m\neq0$. The minimum value of the gravitational potential just due to this distribution of matter is $V_m\sim G\rho_m\overline{d}^2$. Using the same arguments as above, we can calculate the distortion due to the matter in $\delta T/T|_m\sim\beta G\rho_{m0}\overline{d}^3/t$ (β is fudge factor of order unity). Again, the biggest $\delta T/T$ is reached today: $\delta T/T\sim10^{-7}(\overline{d}/20h^{-1}\,\text{Mpc})$.[3]

All other effects, including gravitational distortion at the last photon scattering surface (if $z_f\geq1000\rightarrow\delta T/T|_{\text{LSS}}\sim G\sigma\overline{R_0}a_{\text{LSS}}$, with $a_{\text{LSS}}=10^{-3}$) and effects originating at the phase transition, are comparatively much smaller.

All the values obtained above refer to the distortion originating from a single wall. Even supposing that the phase transition takes place before the photon decoupling, there are only $N\sim R_H/\overline{R_0}\sim3/6\times10^{-2}=50$ walls between us and the last surface of scattering. An evaluation of the $\delta T/T|_m$ due to the matter swept from the walls, which is the biggest distortion, can be obtained multiplying the single-wall distortion by $\sqrt{N}$ and gives $\delta T/T|_m\sim10^{-6}$. For the effects directly related to the domain walls, our values of $\delta T/T$ are, for the same σ, one order of magnitude lower than that calculated in the previous papers, $\delta T/T\sim10\alpha G\sigma\overline{R}^2/t\sim10^{-7}$; this derives from an interwall separation one order of magnitude smaller.[17]

The gravitational interaction of the domain walls with matter is secondary with respect to the sweeping action above described. Taking, for the sake of discussion, the favorable case of one straight infinite wall within the horizon, the peculiar speed gained by the particles due to the gravitational field of the kink would be, after a cosmological time,[15] only of the order $v_m\sim2\pi G\sigma t\sim10^{-4.5}\sim30$ km/s. A complicated network within the horizon scale would certainly give rise to smaller peculiar velocities, due the competing gravitational action of different wall segments. We infer the gravitational effects due to the domain walls themselves are of minor importance, and that the sweeping action onto the dark matter must be done to some Lagrangian term of the kind introduced above.

The self-gravitational effects of the wakes, instead, are important (due to the large density contrast) and should be carefully examined in future simulations that include the baryon component. To first approximation one expects these effects to be confined to the wakes and to the depleted region behind the walls, if $\overline{d}\ll\overline{R}$.

In concluding the discussion we point out that one can also consider late first-order phase transitions in order to achieve our big values for the average interwall distance, even while starting with a much smaller comoving correlation length at the critical temperature. In this way one can remove the lower bound on Δ obtained in this section. Such an analysis is left for future investigation.

VI. CONCLUSION

This offers a framework for future work. We have made the following assumptions: A network of domain walls is established in the primordial Universe through a second-order phase transition; the walls interact with an important gaseous component of the present energy density of the Universe, reflecting elastically all incoming particles regardless of their kinetic energy; the configuration is bound to expand with the background comoving coordinates (up to higher-order corrections).

We reach the following conclusions.

(i) There is a wide range of values for σ, the surface density of the domain walls, such that $\bar{R}_0$, the average interwall distance today, is of the order of the large-scale structure observed for the galaxies.

(ii) The mechanism that generates the fluctuations in the distribution of the dark matter could be related also to the particle Lagrangian, and not just gravitational.

(iii) This suggests that the large-scale structure could indeed form in intimate connection with the presence of the domain walls, although studying the evolution of the fluctuations and the details of the long-distance gravitational effects obtained (see the discussion on the great attractor in Ref. 15) goes beyond the present work.

(iv) Domain walls never come to dominate the energy density of the Universe.

(v) Walls with σ of MeV order and such a small interwall separation ($\bar{R}_0 \sim 100$ Mpc) are not able to distort the microwave background. Also, the effects related to the matter-density fluctuations are small.

Some of the assumptions made to obtain our results may be relaxed, giving rise to the different scenarios we mentioned earlier. Particularly intriguing is the possibility of the wall decoupling mentioned in Sec. III:[18] domain walls may give rise to a spectrum of density perturbations, and at some point decouple and start growing in the way described in the previous work.[4] This paper represents just a first attempt to approach the late phase-transition issue from an angle that could solve some of the problems other investigations have found, and it is meant to stimulate interest in such nonstandard scenarios.

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APPENDIX A

Consider an infinite wall moving along the x axis of a chosen coordinate system, with speed $v>0$. At one side of the wall, particles with speed v_x get reflected, gaining momentum $\Delta p = 2m(v-v_x)$ (particles with $v_x > v$ will not interact with the wall). The momentum distribution of the particles is defined on a (3+3)-dimensional phase space; nevertheless, we are interested in the statistical distribution of the momenta only in the x direction, and we therefore integrate out all other degrees of freedom. In this way we can write, in a very general way, a statistical distribution $f((m/T)^\alpha |v_x|)$ defined so that $B^\alpha \int_{-\infty}^{\infty} f(B^\alpha |v_x|)dv_x = 1$ (where $B \equiv m/T$).[19] The coefficient α depends on the actual original distribution we are considering. For a Boltzmann distribution, $\alpha = \frac{1}{2}$, while for light neutrinos ($m \ll 1$ MeV), $\alpha = 1$.

There are $dN = B^\alpha n(v-v_x) f(B^\alpha |v_x|)dv_x$ interactions per each second and per unit area with momentum exchange Δp (n is the number density of the particles). On the other side of the wall, Δ has opposite sign, so that we can write that the pressure exerted by the gas on the wall is given by

$$P_f = -2mn \int_y^\infty B^{-2\alpha}(y-y_x)^2 f(|y_x|)dy_x + 2mn \int_{-\infty}^{y} B^{-2\alpha}(y-y_x)^2 f(|y_x|)dy_x \; ,$$

where $y = B^\alpha v$ and $y_x = B^\alpha v_x$. The first integral refers to particles having speed $v_x > v > 0$ and hitting the wall from the back, while the second refers to particles hitting the wall from the front.

We now derive, as an example, the form that $f(B^\alpha v)$ assumes in the case of light neutrinos ($m \ll 1$ MeV). We start with the statistical distribution of neutrinos in thermal equilibrium:

$$n = \frac{g}{(2\pi)^3} \int \frac{d^3p}{\exp[(p^2+m^2)^{1/2}/T]+1}, \quad T > T_d \tag{A1}$$

and

$$n = \frac{g}{(2\pi)^3} \int \frac{d^3p}{\exp[(p^2/T^2+m^2/T_d^2)^{1/2}]+1}, \quad T < T_d \; , \tag{A2}$$

where $T \equiv T_d a/a_d$; since $m/T_d \ll 1$, we get

$$n = \frac{g}{(2\pi)^3} \int \frac{d^3p}{\exp(p/T)+1} \; , \tag{A3}$$

at all times.

The probability of finding a particle in an interval p_x, $p_x + dp_x$, of the x component of the momentum is then

$$g(p_x) = \frac{g}{n(2\pi)^3} \int_0^\infty \frac{2\pi p_\perp dp_\perp}{\exp[(p_x^2+p_\perp^2)^{1/2}/T]+1} \; , \tag{A4}$$

where $p_\perp$ is any component of the momentum perpendicular to p_x. Changing variables, we get

$$f(y_x) = \frac{1}{3\zeta(3)} \int_0^\infty \frac{y_\perp dy_\perp}{\exp[(y_x^2+y_\perp^2)^{1/2}]+1} \; , \tag{A5}$$

which is an implicit function of $y_x \equiv m v_x/T$ ($y_\perp \equiv p_\perp/T$). A similar calculation can be performed for a Boltzmann distribution.

APPENDIX B

In this appendix we correct the friction term expressed by Eq. (7) ($y \gg 1$). Here we take into account that particles undergo multiple scatterings from the surface of each wall.

In dealing with this problem, we find it useful to use comoving coordinates, since the comoving speed of the walls turns out to be constant. In comoving coordinates the speed of a free particle is $\dot{r}_p \sim a^{-2}$. In the case $y \gg 1$, after being reflected, particles have a speed double than that of the wall, but since $\dot{r}_p$ increases due to the universal expansion, soon they get scattered again. *A priori* this fact could modify the form of the law of motion of the walls, but this is not the case, as we will see in a moment. Let us give a numerical estimate. Consider a wall moving at constant comoving speed $\dot{r}_0$. At a certain time t_i a particle at rest is reflected and its speed is henceforth given by $\dot{r}_p(t)=2\dot{r}_0(t_i/t)^{4/3}$ (since $a=t^{2/3}$ if $\Omega=1$). The maximum comoving distance $x_{\max}$ from the wall is reached at the time $t_{\max}$, when the wall and particle have equal speed. This yields

$$\dot{r}_p(t_{\max})=\dot{r}_0 \to t_{\max}=1.7t_i \to x_{\max}=0.2\dot{r}t_{\max}=0.2\dot{r}a^{3/2} ,$$

which is roughly as far as any scattered particle can get from the kink.

All the matter in the volume swept by the wall from its formation is contained within a distance $x_{\max}$ in front of the kink, while the total distance traveled by the wall is $\Delta r \sim \dot{r}t_{\max}=\dot{r}a^{3/2}$. Since $\Delta r/x_{\max}\simeq$const, we take the density of the matter in front of the wall to be roughly constant; in this way we get $\rho_{\text{front}}\sim 6\rho_m$ at any time. This means that the initial assumption of constant comoving speed is, at least approximately, self-consistent, substituting K by $K/6$. A more accurate calculation would require a numerical simulation that also takes gravitational effects into account.

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[12] We can do this since both (3) and (6) are valid locally.

[13] This is simply a consequence of the randomness in the initial distribution of the walls, which in our case remain frozen. See the initial conditions in the simulations performed in Ref. 4. See also B. S. Ryden, W. H. Press, and D. N. Spergel, CFA Report No. 3011, 1990 (unpublished).

[14] The assumption that the network is friction dominated from the beginning could be questioned. In fact, the Lagrangian interaction term $\mathcal{L}_{\phi\psi}$ may modify or even prevent the phase transition of the Φ field. To give a rough evaluation of the relative weight of the Lagrangian terms, we can proceed semiclassically. If $f(\phi)_{\max}\langle\bar{\psi}\psi\rangle \sim f(\phi)_{\max}n \ll \sigma\Delta^{-1}$, then $\mathcal{L}_{\phi\psi}$ can be considered small in respect to $\mathcal{L}_{\phi}$. In the simple case that $y \ll 1$ at any $z<z_f$ (which for $z_f \sim 10^3$ corresponds to a particle mass $m>1$ MeV), we have to require $f(\phi)_{\max} \gg mv^2/2$ for the barrier to be high enough to reflect. If $\sigma\Delta^{-1} \gg f(\phi)_{\max}n \gg mnv^2/2$ is satisfied, the phase transition is not affected by $\mathcal{L}_{\phi\psi}$. The requirement $\sigma\Delta^{-1} \gg mnv^2/2$ is met at any $z \ll 15\beta z_f$, so that we can consider the scheme presented as self-consistent. One could also consider Lagrangians with a high degree of symmetry such that the semiclassical estimate of the interaction term is automatically zero. A more complete discussion will be presented in Ref. 18.

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